Chapter 9 Review Name: ____________________________

All answers must be justified with work. No work, no credit!

1. Simplify the following expressions:

A.) $(\sec t \cot t - \csc t \tan t) \sin t \cos t$

$\sin t \cos t \cdot \sec t \cot t - \sin t \cos t \cdot \csc t \tan t$

$\cancel{\sin t \cos t} \cdot \frac{1}{\cos t} \cdot \frac{\cos t}{\sin t} - \cancel{\sin t \cos t} \cdot \frac{1}{\sin t} \cdot \frac{\sin t}{\cos t}$

$\boxed{\cos t - \sin t}$

B.) $\dfrac{1}{2\csc t \cos t - 3\cot t}$

$\dfrac{1}{2 \cdot \frac{1}{\sin t} \cdot \cos t - 3\cot t}$

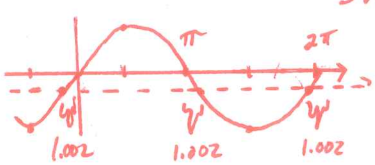
$= \dfrac{1}{2\cot t - 3\cot t}$

$= \dfrac{1}{-\cot t} = \boxed{-\tan t}$

2. What is the smallest positive solution to $2\sin\theta\left(\cos\theta + \dfrac{1}{\sin\theta}\right) = 1.9$? Round to 2 decimal places.

$2\sin\theta\cos\theta + \dfrac{2\sin\theta}{\sin\theta} = 1.9$

$\sin 2\theta + 2 = 1.9$



$\sin 2\theta = -.1$

$2\theta = \sin^{-1}(-.1)$

$2\theta_1 = -.1002$

(negative solution)

$2\theta_2 = \pi - -.1002$ ← Find second angle before dividing by 2.

$2\theta_2 = 3.2418$

$\theta_2 = 1.6209$

$\boxed{\theta_2 = 1.62}$

3. Does $\dfrac{(\cos\theta + \sin\theta)^2 - (\cos\theta - \sin\theta)^2}{\sin 2\theta} = 1$?

NO - GRAPH

$8\sin^2 x - (1 - \sin^2 x) = 5$

$8\sin^2 x - 1 + \sin^2 x = 5$

$9\sin^2 x - 1 = 5$

$9\sin^2 x = 6$

$\sin^2 x = \frac{2}{3}$

$\sin x = \pm\sqrt{\frac{2}{3}}$

$x = \sin^{-1}\left(\sqrt{\frac{2}{3}}\right)$

$\boxed{x = .955}$

$x = \sin^{-1}\left(-\frac{\sqrt{2}}{3}\right)$

$x = -.9553$

AND

$\pi - .9553$

$= \boxed{2.186}$

OR

4. Solve algebraically $8\sin^2 x - \cos^2 x = 5$ for $0 \le t \le \pi$?

$8(1 - \cos^2 x) - \cos^2 x = 5$

$8 - 8\cos^2 x - \cos^2 x = 5$

$-9\cos^2 x + 8 = 5$

$-9\cos^2 x = -3$

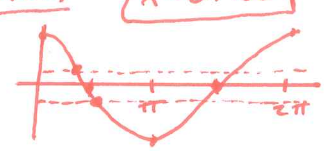
$\cos^2 x = \frac{1}{3}$

$\cos x = \pm\sqrt{\frac{1}{3}}$

$x = \cos^{-1}\left(\sqrt{\frac{1}{3}}\right)$ $x = \cos^{-1}\left(-\sqrt{\frac{1}{3}}\right)$

$x = .9553$ $x = 2.1863$

$\boxed{x = .955}$ $\boxed{x = 2.186}$



5. Prove the identity: $\dfrac{1+\sin\theta}{\cos\theta} = \dfrac{\cos\theta}{1-\sin\theta}$

$\dfrac{1-\sin\theta}{1-\sin\theta} \cdot \dfrac{1+\sin\theta}{\cos\theta}$

$\dfrac{1-\sin^2\theta}{(1-\sin\theta)\cos\theta}$

$\dfrac{\cos^2\theta}{(1-\sin\theta)\cos\theta}$

$\dfrac{\cos\theta}{1-\sin\theta}$ ✓

- OR -

$\dfrac{\cos\theta}{1-\sin\theta} \cdot \dfrac{1+\sin\theta}{1+\sin\theta}$

$\dfrac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta}$

$\dfrac{\cos\theta(1+\sin\theta)}{\cos^2\theta}$

$\dfrac{1+\sin\theta}{\cos\theta}$ ✓

6. Prove the identity: $\cos(2x) = \dfrac{1-\tan^2 x}{1+\tan^2 x}$

Remember, you don't always have to work on the right side of the identity!

$\dfrac{1-\tan^2 x}{\sec^2 x}$

$\dfrac{1-\tan^2 x}{\frac{1}{\cos^2 x}}$

$1-\tan^2 x \div \dfrac{1}{\cos^2 x}$

$(1-\tan^2 x)\cos^2 x$

$\cos^2 x - \dfrac{\sin^2 x}{\cancel{\cos^2 x}} \cdot \cancel{\cos^2 x}$

$\cos^2 x - \sin^2 x$

$\cos 2x$ ✓

7. If $s(x) = \sin(3x) + \sin(2x)$, then $s(x)$ can also be written in the form
$s(x) = \underline{\ 2\ }\sin(\underline{\ 2.5\ }x)(\cos\underline{\ \tfrac{1}{2}\ }x)$.

$2\sin\left(\dfrac{3x+2x}{2}\right)\cos\left(\dfrac{3x-2x}{2}\right)$

$2\sin\left(\frac{5x}{2}\right)\cos\left(\frac{x}{2}\right)$

$2\sin(2.5x)\cos\left(\frac{1}{2}x\right)$

8. Using the sum or difference formulas, $6\sin 3t - 5\cos 3t = \underline{7.8102}\sin(\underline{3}\ t + \underline{-.6947})$. Round all answers to 4 decimal places.

$a_1 = 6 \quad a_2 = -5$

$\phi = \tan^{-1}\left(\frac{-5}{6}\right)$

$A = \sqrt{6^2 + (-5)^2}$

$= \sqrt{61}$

$\phi = -.6947 \ (Q4)$

$\cos\phi = \frac{6}{\sqrt{61}}$

$= \sqrt{61}\sin(3t + -.6947)$

$= 7.8102\sin(3t - .6947)$

$\sin\phi = \frac{-5}{\sqrt{61}}$

Q4

9. Find the smallest value of t such that $t > 0$ and $\cos(10t) + \cos(9t) = 0$.

$2\cos\left(\frac{10t+9t}{2}\right)\cos\left(\frac{10t-9t}{2}\right) = 0$

$2\cos\left(\frac{19}{2}t\right)\cos\left(\frac{t}{2}\right) = 0$

$2\cos\left(\frac{19}{2}t\right) = 0 \qquad \cos\left(\frac{t}{2}\right) = 0$

$\cos\left(\frac{19}{2}t\right) = 0 \qquad \frac{t}{2} = \cos^{-1} 0$

$\frac{19}{2}t = \cos^{-1} 0 \qquad \frac{t}{2} = \frac{\pi}{2}$

$\frac{19}{2}t = \frac{\pi}{2} \qquad t = \frac{\pi}{2} \cdot 2$

$t = \frac{\pi}{2} \cdot \frac{2}{19} \qquad t = \pi$

$\boxed{t = \frac{\pi}{19}}$

10. Find the exact value of sin 165 degrees.

$\sin(120^\circ + 45^\circ) = \sin 120^\circ \cos 45^\circ + \cos 120^\circ \sin 45^\circ$

$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + -\frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$\boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$

11. Find a formula for a deer population which oscillates over a 6 year period between a low of 1000 in year $t = 0$ and a high of 3500 in year $t = 3$.

Reflected cosine

Amplitude $= \frac{3500-1000}{2}$

$A = 1250$

Midline: $y = \frac{3500+1000}{2}$

$y = 2250$

Period $= 6$

$6 = \frac{2\pi}{B}$

$6B = 2\pi$

$B = \frac{2\pi}{6}$

$B = \frac{\pi}{3}$

$f(t) = -1250\cos\left(\frac{\pi}{3}t\right) + 2250$

12. The deer population in a state park is modelled by $f(t) = 70\sin\left(\frac{\pi t}{6}\right) + 220$ where t is the number of months since January 1, 2005. Evaluate $f(6) - f(3)$ and interpret the result. Round to the nearest whole number.

$f(6) - f(3)$

$70\sin\left(\frac{\pi \cdot 6}{6}\right) + 220 - \left(70\sin\left(\frac{3\pi}{6}\right) + 220\right)$

$70\sin\pi + 220 - \left(70\sin\frac{\pi}{2} + 220\right)$

$70(0) + 220 - (70(1) + 220)$

$220 - 290$

$\boxed{-70}$

The deer population decreases by 70 from April to July.

13. A mass attached to a spring moves horizontally on a frictionless track. Its displacement from the rest position at time t is given by $x = 0.2\cos(5t)$. What is the furthest distance from the rest position that the mass will achieve? The displacement is measured in meters.

Max value of cosine is 1, so max displacement will be .2 meters.